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Aspects of Recording Synthetic Array Data

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Sensing and Information Subdivision

Prepared for COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California

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Communications Systems Technical Report No. 2

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ABSTRACT

The size and resolution of the desired map play a direct role in specifying the recording requirements of synthetic array data. A detailed investigation of the system indicates the required bandwidths, demodulations, and stabilities of intermediate processing which are required. Several approaches are presented in this paper with a corresponding analysis to indicate the consequences of their assumptions and the theoretical bounds on their operation.

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ASPECTS OF RECORDING SYNTHETIC ARRAY DATA

The size and resolution of the desired map play a direct role in specifying the recording requirements of synthetic array data. A detailed investigation of the system indicates the required bandwidths, demodulations, and stabilities of intermediate processing which are required. Several approaches are presented in this paper with a corresponding analysis to indicate the consequences of their assumptions and the theoretical bounds on their operation.

1.0 SINGLE TARGETS

An isolated target at a distance R is assumed to return a transmitted signal with a delay ζ

$$\zeta = \frac{2R}{c}$$

where c is the velocity of propagation. The loss in amplitude of the reflected signal is a function of the antenna gain, the distance R, the wavelength λ, and the reflectance γ of the target. The return signal f_2 in terms of the transmitted signal f_1 is

$$f_2(t) = \left(\frac{G}{4\pi R^2}\right)^{\frac{1}{2}} \gamma \left(\frac{1}{4\pi R^2}\right)^{\frac{1}{2}} \left(\frac{G\lambda^2}{4\pi}\right)^{\frac{1}{2}} f_1\left(t - \frac{2R}{c}\right)$$

The first factor produces the voltage density at the target. The second factor γ is the effective length of the target in the direction of the antenna. The third factor is the effect of dispersion of the reradiated signal, and the fourth factor is the effective length of the antenna.

In terms of systems, it is convenient to consider the isolated target as acting as a linear transfer function as in Figure 1.

Figure 1. An Isolated Target Acting as a Linear Transfer Function

The impulse response $h(\zeta)$ of a target at distance R is then

$$h(\zeta) = \frac{4\,G\lambda\gamma}{(4\pi)^{3/2}\,c^2\zeta^2}\,\delta\left(\zeta - \frac{2R}{c}\right)$$

where $\delta(\zeta)$ is a unit impulse function.

1.1 Distance

The precise computation of the distance to a particular target having coordinates (r, ϕ) is needed in the expression for the impulse response of the ground targets. From Figure 2, the square of the distance between the

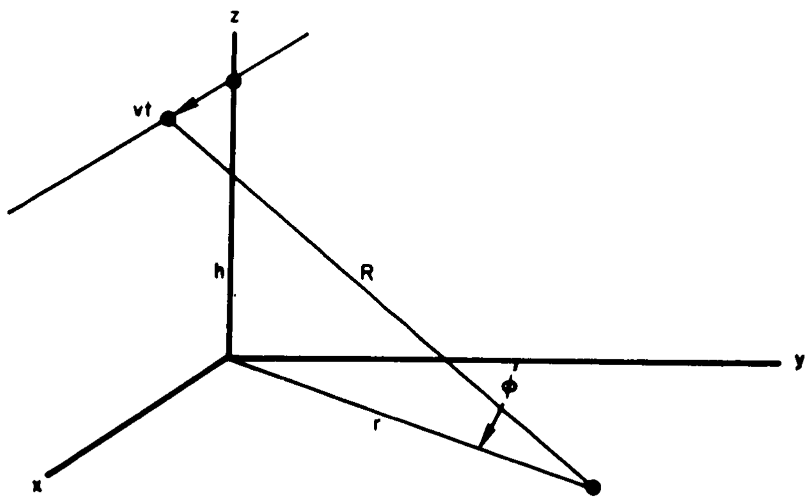

Figure 2. Geometry of Distance Calculations

coordinate (r, ϕ) and a vehicle at altitude h moving with velocity v is

$$R^2(t, r, \phi) = h^2 + r^2 \cos^2\phi + (r \sin \phi - vt)^2$$
$$= R_o^{\,2} - 2vt\, r \sin \phi + (vt)^2$$

where $R_o^{\,2} = h^2 + r^2$.

The use of three terms of a Taylor series expansion for the square root produces the relation

$$R(t, r, \phi) \approx R_o - \frac{vt\, r \sin \phi}{R_o} + \frac{(vt)^2}{2R_o^{\,3}} (h^2 + r^2 \cos^2 \phi).$$

In this particular nomenclature the distance R is essentially R_o. An estimate of the incremental change in distance R between transmission and reception from the coordinate (r, ϕ) is

$$\text{Incremental change in R} = 2\frac{v}{c}\, r \sin \phi$$

For orbital velocities and 2,000 feet for $r \sin \phi$, the change is 1/10 ft. This change is insignificant in terms of geometry but quite significant in producing a Doppler shift in frequency and in processing the subsequent phase data.

As an example of these effects, the transmission of the short pulse u(t) of high frequency

$$f_1(t) = u(t)\, e^{j\omega t}$$

produces the return $f_2(t)$

$$f_2(t) = \int f_1(t-\sigma)\, h(\sigma)\, d\sigma$$

$$= \int u(t-\sigma)\, e^{j\omega(t-\sigma)} \frac{4\, G\lambda\gamma}{(4\pi)^{3/2}\, c^2 \sigma^2}\, \delta\left(\sigma - \frac{2R}{c}\right) d\sigma$$

$$= \frac{G\lambda\gamma}{(4\pi)^{3/2}\, R^2}\, u\left(t - \frac{2R}{c}\right) e^{j\omega t - \frac{4\pi}{\lambda}\left[R_o - \frac{vtr\ \sin\phi}{R_o} + \frac{(vt)^2}{2R_o^{\ 3}}(h^2 + r^2 \cos^2\phi)\right]}$$

where the operation of taking the real part of the exponentials is implied. The effect of the small variation in distance is to introduce a Doppler shift in the high frequency carrier of $\frac{2vr\ \sin\phi}{\lambda R_o}$ cycles per second.

1.2 Multiple Targets

In actual practice the returning signal from a transmitted signal is made up of reflections of many targets scattered over the ground. In a linear system, the impulse responses of the targets sum to form a composite impulse response characterizing the reflectance of the particular terrain being observed.

In a sufficiently small time interval $\Delta\zeta$, the integral of the composite impulse response $h(\zeta)$ is

$$\int_{\Delta\zeta} h(\zeta)d\zeta = h(\zeta)\Delta\zeta = \frac{\lambda}{(4\pi)^{3/2}\, R^2} \sum_i G(\phi_i)\gamma(r, \phi_i),$$

where the summation is over those targets for which

$$\zeta = \frac{2}{c} R = \frac{2}{c}\sqrt{r^2 + h^2}$$

within the limits of the time interval $\Delta\zeta$.

A more convenient expression is

$$h(\zeta)\Delta\zeta = \frac{\lambda}{(4\pi)^{3/2} R^2} \int G(\phi)\, \gamma_o(r, \phi)\, r d\phi \frac{h}{R}\Delta r,$$

where the integral is over the angle ϕ, and the reflectance $\gamma_o(r, \phi)$ is the average length per unit area. Further simplification results from using the expression

$$R\Delta\zeta = R\frac{2}{c}\Delta R = \frac{2}{c} r\Delta r$$

so that the composite impulse function is

$$h(\zeta) = \frac{2\lambda h}{(4\pi)^{3/2} c\, \zeta^2} \int G(\phi)\, \gamma_o(r, \phi)\, d\phi,$$

where $$r = \sqrt{\left(\frac{c}{2}\zeta\right)^2 - h^2}\ .$$

and $$h(\zeta) = 0 \qquad \zeta < \frac{2h}{c}.$$

A particular example is the characterization of the targets at a range r_o:

$$\gamma_o(r, \phi) = \gamma_o(\phi)\, \delta(r - r_o).$$

The response to the transmitter high frequency pulse

$$f_1(t) = u(t)\, e^{j\omega t}$$

is then

$$f_2(t) = \int f_1(t-\sigma)\, h(\sigma)\, d\sigma$$

$$= \int u(t-\sigma)\, e^{j\omega(t-\sigma)} d\sigma\, \frac{2\lambda h\, \delta(r-r_o)}{(4\pi)^{3/2}\, c\sigma^2} \int G(\phi)\, \gamma_o(\phi) d\phi$$

$$= \frac{\lambda h}{(4\pi)^{3/2}\, R_o^{\,2}}\, u\left(t - \frac{2R_o}{c}\right) e^{j\omega t} \int G(\phi)\gamma_o(\phi) e^{\frac{-j4\pi}{\lambda} R(t, r_o, \phi)}\, d\phi$$

where $R_o^{\,2} = h^2 + r_o^{\,2}$.

The integral in the expression for the return signal sums the targets at range r_o, each signal modifying the high frequency $e^{j\omega t}$ by its appropriate Doppler shift.

Based on the assumption of a symmetrical antenna pattern $G(\phi)$, the spectrum width of the Doppler Δf_d is twice the Doppler shift f_d at the edge of the antenna pattern ϕ_d:

$$\Delta f_d = 2f_d = \frac{2}{2\pi} \frac{d}{dt} \frac{4\pi}{\lambda} R(t, r_o, \phi_d)$$

$$= \frac{4vr \sin \phi_d}{R_o \lambda}$$

2.0 DEMODULATION

The most common method of demodulating the high frequency return is by using an offset frequency ω_o. Product demodulation of the targets at a range r_o with

$$f_3(t) = 2 \cos (\omega - \omega_o)t,$$

as in Figure 3, produces the signal

$$f_4(t) = \frac{\lambda h}{(4\pi)^{3/2} R_o^2} u\left(t - \frac{2R_o}{c}\right) e^{j\omega_o t} \int G(\phi)\, \gamma_o(\phi) e^{-j\frac{4\pi}{\lambda} R(t, r_o, \phi)} d\phi$$

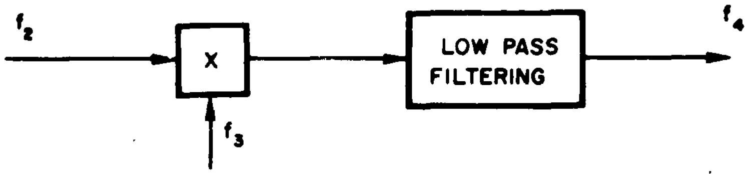

Figure 3. Product Demodulation with an Offset Frequency, ω_o, to Preserve the Doppler Information

It is this signal which is normally recorded and subsequently processed into a map of the reflectance $\gamma_o(r, \phi)$.

The recording can be made either optically on film or magnetically on a tape recorder. In the case of optical recording, the film acts as a defraction grating with an offset collimation corresponding to the offset frequency ω_o. In the case of magnetic tape recording, the bandwidth is principally determined by the sum of the base bandwidth of the transmitted pulse and the offset frequency ω_o.

Considerable accuracy is needed in the frequency of the demodulating signal. Based on the limit that the phase variation must be limited to less than π radians

$$\Delta\phi \leq \pi$$

over the build-up of the array, the frequency stability of the product demodulation is bounded by the expression

$$\frac{f}{\Delta f} \geq \frac{c}{v} \frac{R}{\delta_x} .$$

Figure 4 is a plot of this stability for orbital velocities.

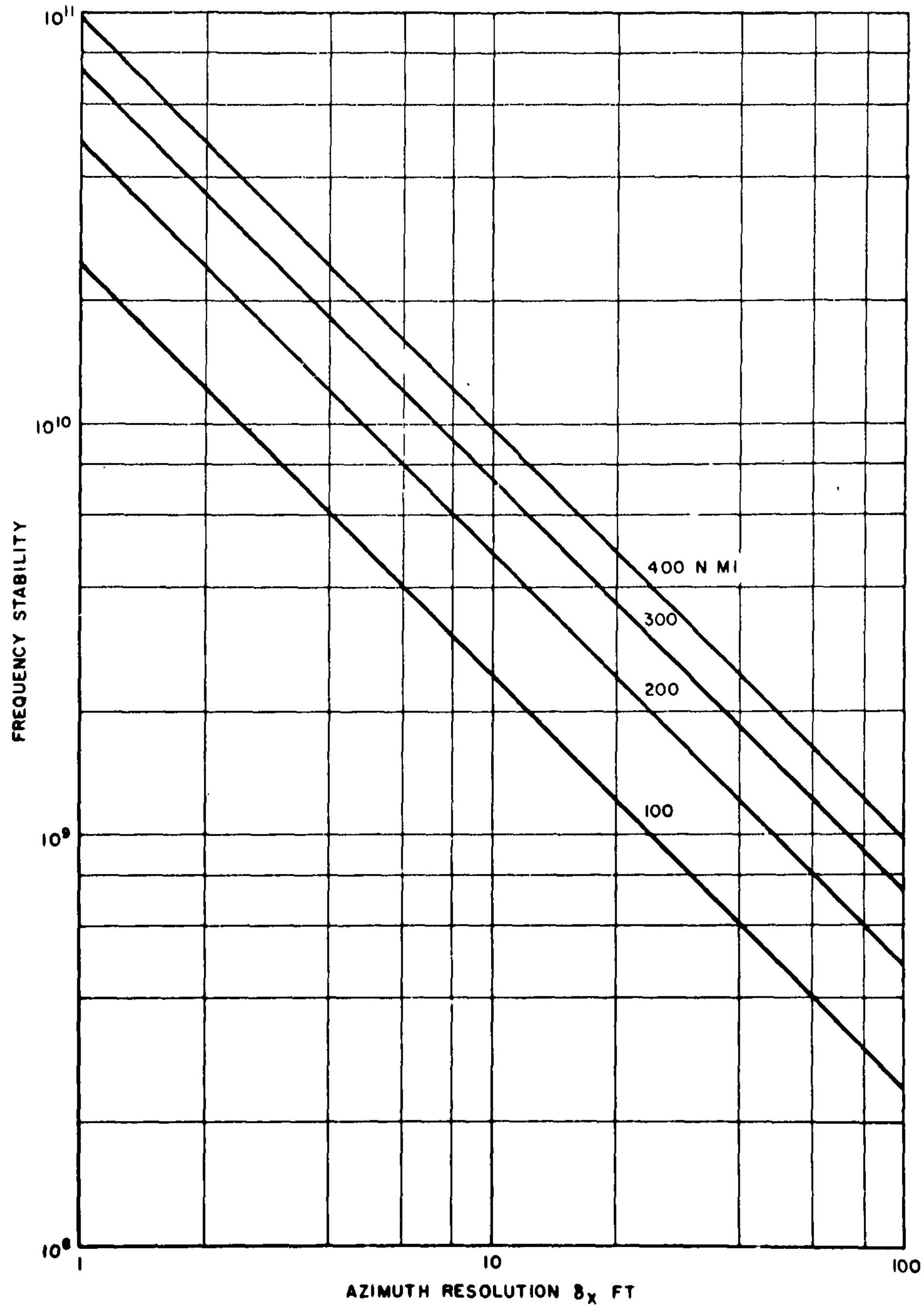

Figure 4. Frequency Stability of a Synthetic Array at Orbital Velocities

$$\frac{f}{\Delta f} \geq \frac{c}{v} \frac{R}{\delta_x}$$

3.0 RANGE RESOLUTION BANDWIDTH

The range resolution of the system comes from the ability of the return $u\left(t-\frac{2R}{c}\right)$ to represent in amplitude a sequence of independent measurements. A measure of this representation ability is the rise-time δ_T of the function. When a function is passed through a low-pass filter such as that of Figure 5, the rise-time of the resulting function is normally limited to the reciprocal of twice the base bandwidth W of the network.

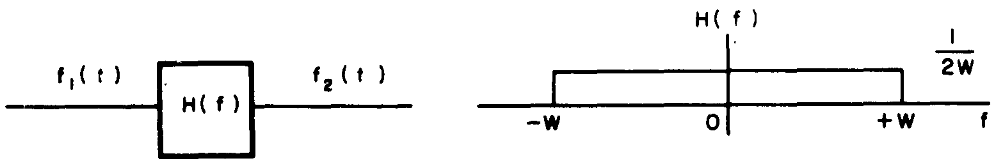

Figure 5. Passing a Function Through a Network Having a Base Bandwidth W

The rise-time δ_T of a network having a base bandwidth W cycles per second can be computed by first finding the impulse response of the network and then finding the response to a unit step function u(t). The impulse response is

$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{i2\pi ft} df$$

$$= \int_{-W}^{W} \frac{1}{2W}\, e^{i2\pi ft} df$$

$$= \frac{\sin 2\pi Wt}{2\pi Wt}$$

The response to the unit step u(t) is

$$f_2(t) = \int_{-\infty}^{\infty} u(t-\zeta)\, h(\zeta)\, d\zeta$$

$$= \int_{-\infty}^{t} \frac{\sin 2\pi W\zeta}{2\pi W\zeta}\, d\zeta$$

$$= \frac{1}{2\pi W} \int_{-\infty}^{2\pi W t} \frac{\sin x}{x}\, dx$$

Figure 6 is a plot of this response.

Since the final value of response is

$$f_2(\infty) = \frac{1}{2W}$$

with the slope at the origin

$$f_2'(0) = 1,$$

the rise-time is essentially

$$\delta_\tau = \frac{1}{2W}.$$

The implications are that in order to resolve a sequence of independent amplitudes, the network must have a base bandwidth W bounded by the relation

$$W \geq \frac{1}{2\delta_\tau}.$$

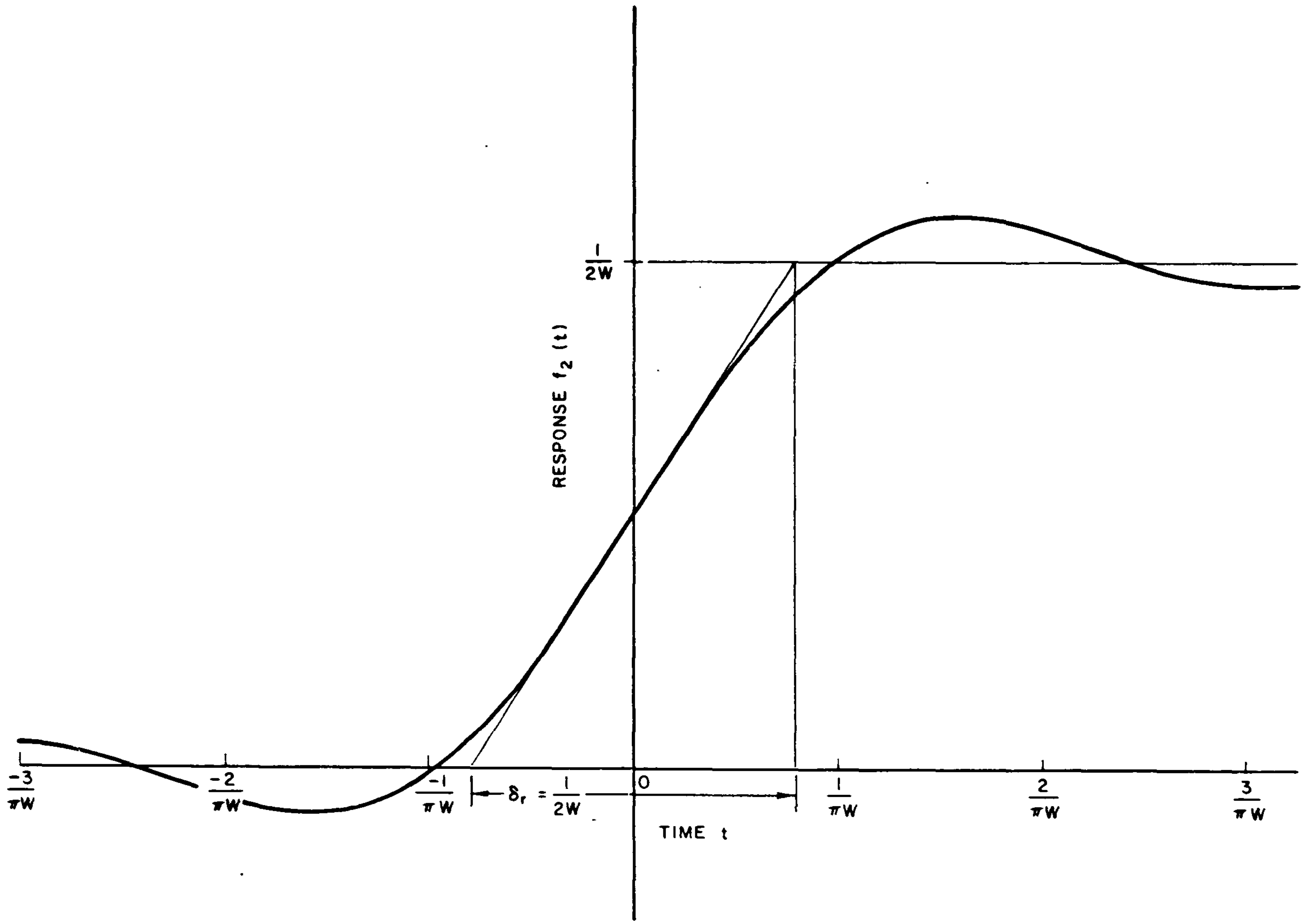

Figure 6. Response to a Unit Step Passed through a Network Having Base Bandwidth W

Use of the range resolution bound

$$\frac{1}{\delta_\tau} \geq \frac{c}{2\,\delta_r \cos\psi}$$

produces the bound on the base bandwidth

$$W \geq \frac{c}{4\,\delta_r \cos\psi}$$

where ψ is the depression angle from the horizon. This relation is plotted in Figure 7.

In terms of lowpass-to-bandpass equivalence, the high frequency bandwidth of the radar return is bounded by the reciprocal of the rise-time.

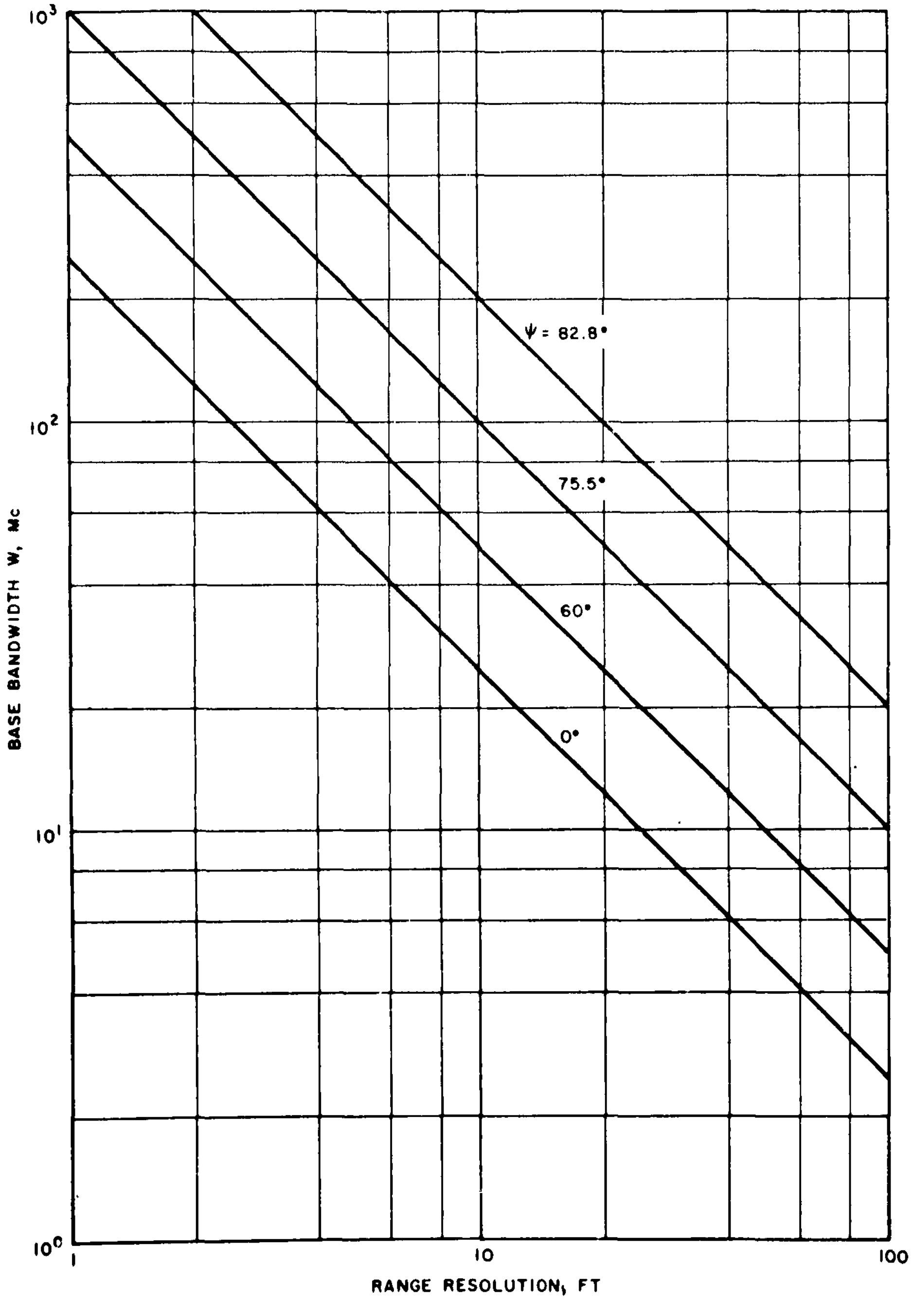

Figure 7. Base Bandwidth in terms of Range Resolution and Depression Angle ψ

$$W \geq \frac{c}{4\delta_r \cos \psi}$$

4.0 SAMPLING RANGE RETURNS

The mechanization of the azimuth resolution results from a sampling of the phase variations of a target at regular intervals in the trajectory. The range R to a particular target is computed from the formula

$$R^2(t) = R_o^2 + v^2t^2$$

It follows that

$$R(t) \approx R_o + \frac{v^2t^2}{2R_o}$$

The sample at the time corresponding to this range is then

$$f_5(t) = \frac{\lambda h}{(4\pi)^{3/2}\, R^2}\, u\left(t - \frac{2R}{c}\right) \int G(\phi)\, \gamma_o(\phi) d\phi\, e^{j\omega_o t - j\frac{4\pi}{\lambda} R(t)}$$

where the last term is a constant phase term

$$-\frac{4\pi}{\lambda} R_o$$

and a linearly varying radian frequency

$$\omega_o - \frac{4\pi\, v^2 t}{\lambda R_o}$$

varying through twice the Doppler shift $\omega_o + \omega_d$ to $\omega_o - \omega_d$. At the edges of the antenna pattern

$$|X| = |vt| = \frac{\omega_d}{2\pi} \frac{\lambda R_o}{2v} = \frac{\omega_d}{2\pi} \frac{L\delta_x}{v}$$

where L is the total build-up length and δ_x is the azimuth resolution.

A straight-forward application of the sampling theorem would imply a sampling rate of twice the spectrum width Δf_d, or four times the Doppler frequency f_d. On the other hand, the technique of breaking the signal into the in-phase $f_i(t)$ and quadrature-phase $f_g(t)$ components

$$f(t) = f_i(t) \cos \omega_d t + f_g(t) \sin \omega_d t$$

as in Figure 8, implies a sampling rate equal to twice the Doppler frequency f_d.

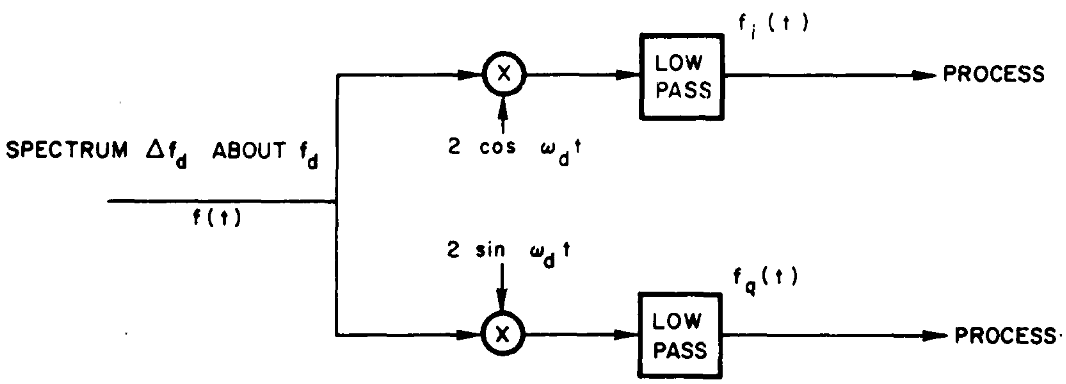

Figure 8. Sampling the In-Phase and Quadrature-Phase of a Doppler Spectrum

A second method of obtaining the in-phase and quadrature-phase components is through a dual sampling of the demodulated returns. Sampling the return at t_o and $t_o + \frac{1}{4f_o}$ essentially produces the cosine and the sine. The restriction on this type of sampling is that the envelope of the signal remains essentially constant. By use of the bound that the sampling should be within $\frac{1}{2}$ the resolution time δ_τ,

$$\frac{1}{4f_o} < \frac{\delta_\tau}{2}$$

and range resolution bound

$$\frac{1}{\delta_\tau} \geq \frac{c}{2\,\delta_r \cos\psi} \quad ,$$

the offset frequency is bounded by the expression

$$f_o \geq \frac{c}{4\,\delta_r \cos\psi} \quad .$$

This bound is the base bandwidth of the high frequency pulse. The recording bandwidth is then necessarily twice the base bandwidth given in Figure 7.

The bound on sampling places a bound on the pulse repetition frequency of the system

$$\text{prf} \geq \Delta f_d = \frac{v}{\delta_x}\,\frac{X}{L}$$

where X is the illuminization breadth and L is the build-up length. Use of the relation

$$\mathrm{prf} = \frac{v}{\Delta}$$

where Δ is the element spacing of the array produces the bound

$$\Delta \leq \delta_x \frac{L}{X}$$

5.0 IN-PHASE AND QUADRATURE-PHASE PROCESSING

The processing of data is normally accomplished by a technique known as match filtering where a target is recognized by correlating a normalized target against the data. This approach assumes that the system is linear and that the interfering noise and data are broadband.

Detection of a target represented by the signal

$$u \cos\left(-\frac{4\pi}{\lambda} R_o + \omega_d t - \frac{2\pi}{\lambda} \frac{v^2 t^2}{\lambda R_o}\right)$$

is complicated by the lack of knowledge of the range R_o. Woodward* has shown that a sufficient technique in Gaussian noise and detection with arbitrary phase is that shown in Figure 9.

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*P. M. Woodward, "Probability and Information Theory with Application to Radar," New York, Pergamon, 1953.

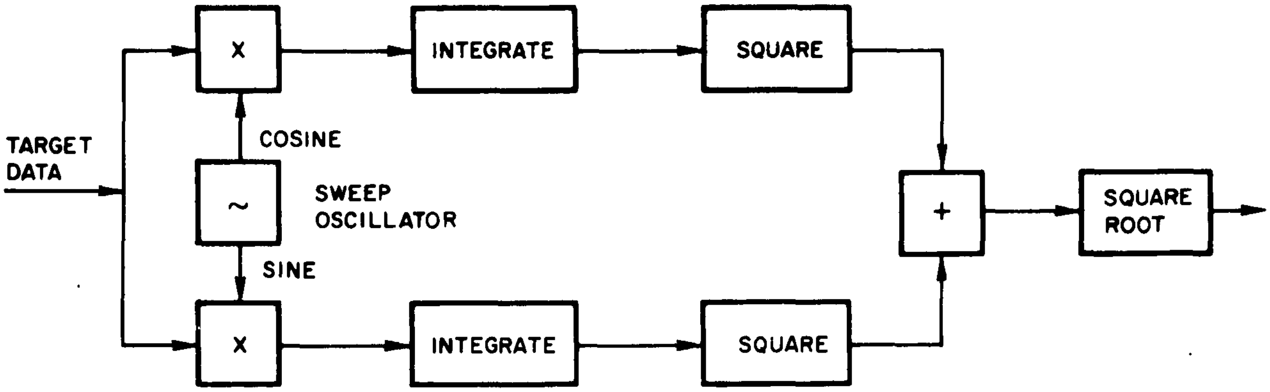

Figure 9. The Detection of a Target with Arbitrary Phase

The target data is multiplied by

$$2\cos\left(\omega_d t - \frac{2\pi}{\lambda}\frac{v^2t^2}{\lambda R_o}\right) \quad \text{and} \quad 2\sin\left(\omega_d t - \frac{2\pi}{\lambda}\frac{v^2t^2}{\lambda R_o}\right)$$

Subsequent integration essentially drops the sum terms and produces the two outputs

$$\Delta T\ u\cos\left(\frac{4\pi}{\lambda}\right)R \quad \text{and} \quad \Delta T\ u\sin\left(\frac{4\pi}{\lambda}\right)R$$

where ΔT is the interval of integration. Squaring these outputs, adding them, and taking the square root produces the estimate $\Delta T\ u$ of the target.

Undesired targets which are displaced in time are essentially lost in the integration process. The principal difficulty is one of obtaining the correct frequency sweep rate corresponding to the continuously varying range. This problem is usually referred to as focussing.

6.0 PRE-SUMMING SYNTHETIC ARRAY DATA

An interesting technique in the compression of synthetic array data is based on the fact that the phase variation from sample to sample is sufficiently small to permit a summation of the data without making the appropriate phase corrections to focus the synthetic beam.

6.1 Incremental Phase

The geometry of Figure 10 indicates that the phase between a transmitted and received signal reflecting off a target is

$$\phi(x) = \frac{4\pi}{\lambda} \sqrt{R^2 + x^2}$$

where R is the distance of closest approach and λ is the wavelength

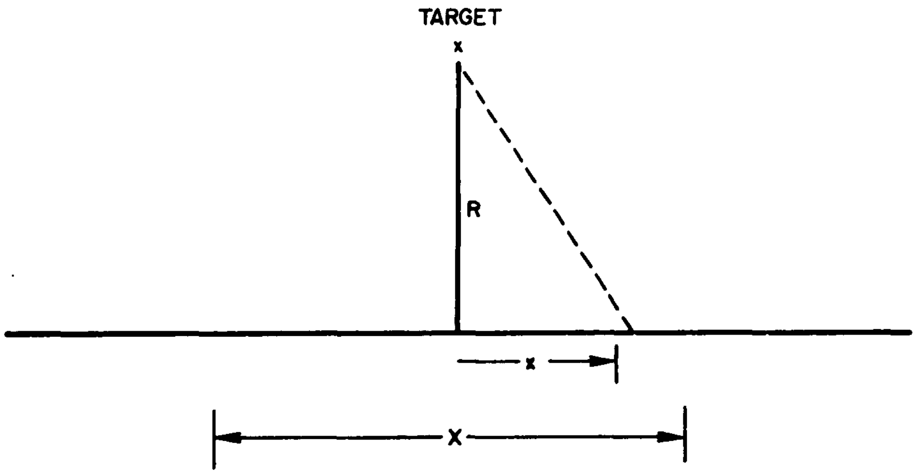

Figure 10. Geometry of Phase Variations

The slope of the phase variation is then

$$\phi'(x) = \frac{4\pi}{\lambda} \frac{x}{\sqrt{R^2 + x^2}}$$

so that the phase change between samples separated by a distance Δ is

$$\Delta\phi = \frac{4\pi}{\lambda} \frac{x}{\sqrt{R^2 + x^2}} \Delta$$

$$\approx \frac{4\pi\, x\, \Delta}{\lambda R} \quad .$$

The beamwidth of a synthetic array of length L is given by the relation

$$B_s = \frac{\lambda}{2L}$$

so that the resolution δ_x is obtained by the formula

$$\delta_x = RB_s = \frac{\lambda R}{2L} \quad .$$

The incremental phase change at the edge of the illumination ($x = \frac{X}{2}$) is then

$$\Delta\phi = \pi \frac{\Delta}{\delta_x} \frac{X}{L} \quad .$$

The summation of two successive samples without a correction in-phase would require that

$$\Delta\phi \leq \frac{\pi}{2} \quad \text{or} \quad \Delta_s \leq \frac{1}{2}\, \delta_x \frac{L}{X} \quad .$$

The Doppler spectrum of frequencies obtained from sampling the target of Figure 10 can be obtained by the time derivative of the phase. The Doppler shift in frequency is then

$$f_d = \frac{1}{2\pi}\frac{d}{dt}\,\phi$$

$$= \frac{1}{2\pi}\frac{4\pi}{\lambda}\,\frac{x}{\sqrt{R^2 + x^2}}\,v$$

$$\approx \frac{2xv}{\lambda R}$$

The Doppler spectrum is then

$$\Delta f_d = \frac{2Xv}{\lambda R} = \frac{v}{\delta_x}\frac{X}{L}$$

The use of in-phase and quadrature-phase recording and processing requires that the pulse repetition rate be bound by the width of the Doppler spectrum

$$\text{prf} \geq \Delta f_d \quad .$$

The sampling is necessarily the sampling of the radar system, so that in terms of displacement of the antenna Δ between samples .

$$\text{prf} = \frac{v}{\Delta} \geq \frac{v}{\delta_x}\frac{X}{L} \quad .$$

This bound requires that

$$\Delta \leq \delta_x \frac{L}{X} \quad .$$

A comparison of the upper bound on the spacing Δ of the array elements is given in Figure 11.

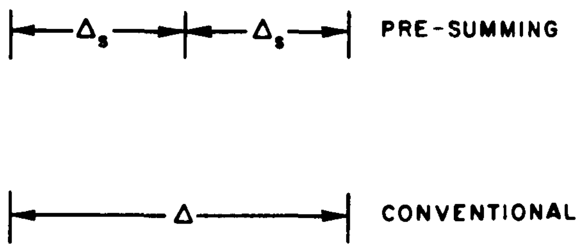

Figure 11. Spacing of Samples in the Construction of a Synthetic Array

In most cases, it is desirable to operate as close to these upper bounds as is practical. A comparison of the bounds indicates that in order to sum adjacent samples, it is necessary to operate the system at twice the speed of the conventional system. In essence, pre-summing requires that twice the data of a conventional system be obtained. This data must then be reduced by a factor of two to produce the conventional results.

6.2 Artificial Antenna Patterns

A second disadvantage to pre-summing adjacent samples is the creation of an artificially scalloped antenna pattern within the limits of radiation.

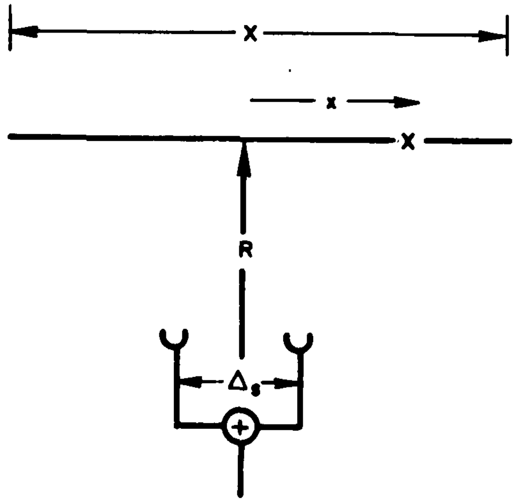

Figure 12. A Geometric Model of Pre-Summing

The effective antenna pattern is obtained from the geometry of Figure 12, where the phases of the two signals to be added are

$$\phi_1 = \frac{4\pi}{\lambda}\sqrt{R^2 + \left(x + \frac{\Delta_s}{2}\right)^2} \approx \frac{4\pi}{\lambda}\sqrt{R^2 + x^2} + \frac{2\pi}{\lambda}\frac{x\Delta_s}{R}$$

and

$$\phi_2 = \frac{4\pi}{\lambda}\sqrt{R^2 + \left(x + \frac{\Delta_s}{2}\right)^2} \approx \frac{4\pi}{\lambda}\sqrt{R^2 + x^2} - \frac{2\pi}{\lambda}\frac{x\Delta_s}{R} \quad .$$

The resultant summation is then

$$e^{j(\omega t+\alpha)}\left\{e^{j\phi_1} + e^{j\phi_2}\right\}$$

$$\approx 2e^{j\left(\omega t+\alpha+\frac{4\pi}{\lambda}\sqrt{R^2 + x^2}\right)} \cos\frac{2\pi}{\lambda}\frac{x\Delta_s}{R} \quad .$$

The first null in the addition occurs where the argument of the cosine is $\pi/2$ or where

$$\theta \approx \frac{x}{R} = \frac{\lambda}{4\Delta_s} \quad .$$

Normally, the physical antenna is made with the largest possible beamwidth to produce the smallest dimensions. The first synthetic ambiguity of a conventional synthetic array occurs at

$$\theta \approx \frac{\lambda}{2\Delta} \quad .$$

Thus the pre-summing null falls within the useful beam of the synthetic aperture. An alternate approach would be to use half the sampling space,

$$\Delta_s = \frac{1}{2}\Delta \quad ,$$

so that the pre-summing null falls on the first synthetic ambiguity. However, this procedure produces twice the data which is subsequently reduced by a factor of two to produce the same amount of data obtainable in a conventional system. The advantage of pre-summing seems to be non-existent.

7.0 GROUND RETURN RANGE-GATING

One method of reducing the severe antenna sidelobe requirements in a synthetic array is to range-gate out the undesirable energy from the ground directly below the vehicle. The return is then double-recorded to effectively "stretch out" the range interval to the period of the pulse repetition.

From the geometry of Figure 13, the period of the undesired and desired returns is

$$\Delta\tau' = \frac{2\Delta R' \cos\psi}{c}$$

where ψ is the depression angle and c is the velocity of propagation. It necessarily follows that the pulse repetition rate is bounded such that

$$prf' \leq \frac{1}{\Delta\tau'} = \frac{c}{2\Delta R' \cos\psi} \quad .$$

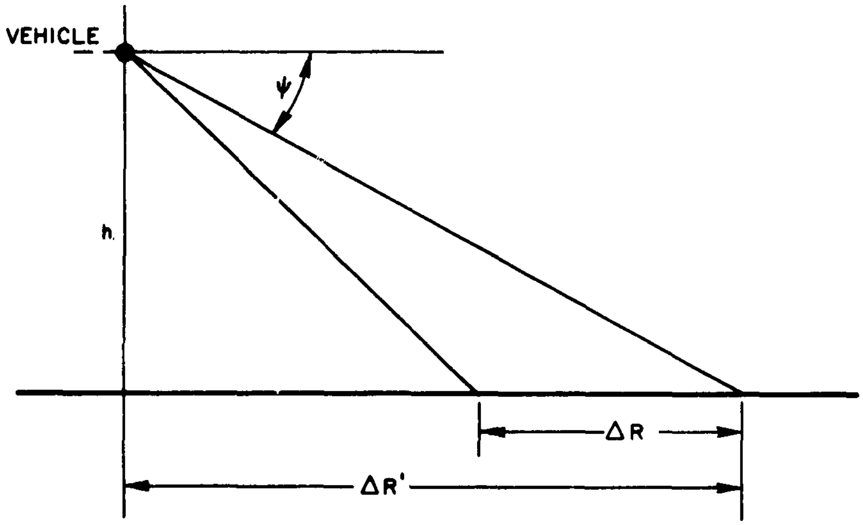

Figure 13. Ranging Geometry of a Vehicle at an Altitude h

The use of the azimuth-element-rate bound

$$\frac{v}{\delta_x'} \leq \text{prf}$$

produces a lower bound on the azimuth resolution

$$\delta_x' \geq 2\,\frac{v}{c}\,\Delta R' \cos \psi \quad .$$

In the case where range-gating is not used, the period associated with mapping the strip ΔR of Figure 13 is

$$\Delta\tau = \frac{2\Delta R \cos \psi}{c}$$

so that the pulse repetition is bounded as

$$\mathrm{prf} \leq \frac{1}{\Delta\tau} = \frac{c}{2\Delta R \cos\psi} \quad .$$

The azimuth resolution is thus bounded as

$$\delta_x \geq 2\,\frac{v}{c}\,\Delta R \cos\psi \quad .$$

The ratio of the two bounds on the azimuth resolution δ_x'/δ_x is a measure of the degradation in theoretical azimuth resolution resulting from range-gating. In terms of the altitude h of the vehicle,

$$\text{Degradation} = \frac{\Delta R'}{\Delta R} = \frac{h}{\Delta R \tan\psi}$$

where ΔR is the width of the strip to be mapped.

Figure 14 is a plot of this function for a strip width of 50 n mi. From this figure it can be seen that for altitudes above 200 n mi, it is necessary to use depression angles above 60° to keep the degradation below a factor of 2. It is reasonable to expect degradations of 4 to 10 and higher for higher altitudes and small widths of the map strip.

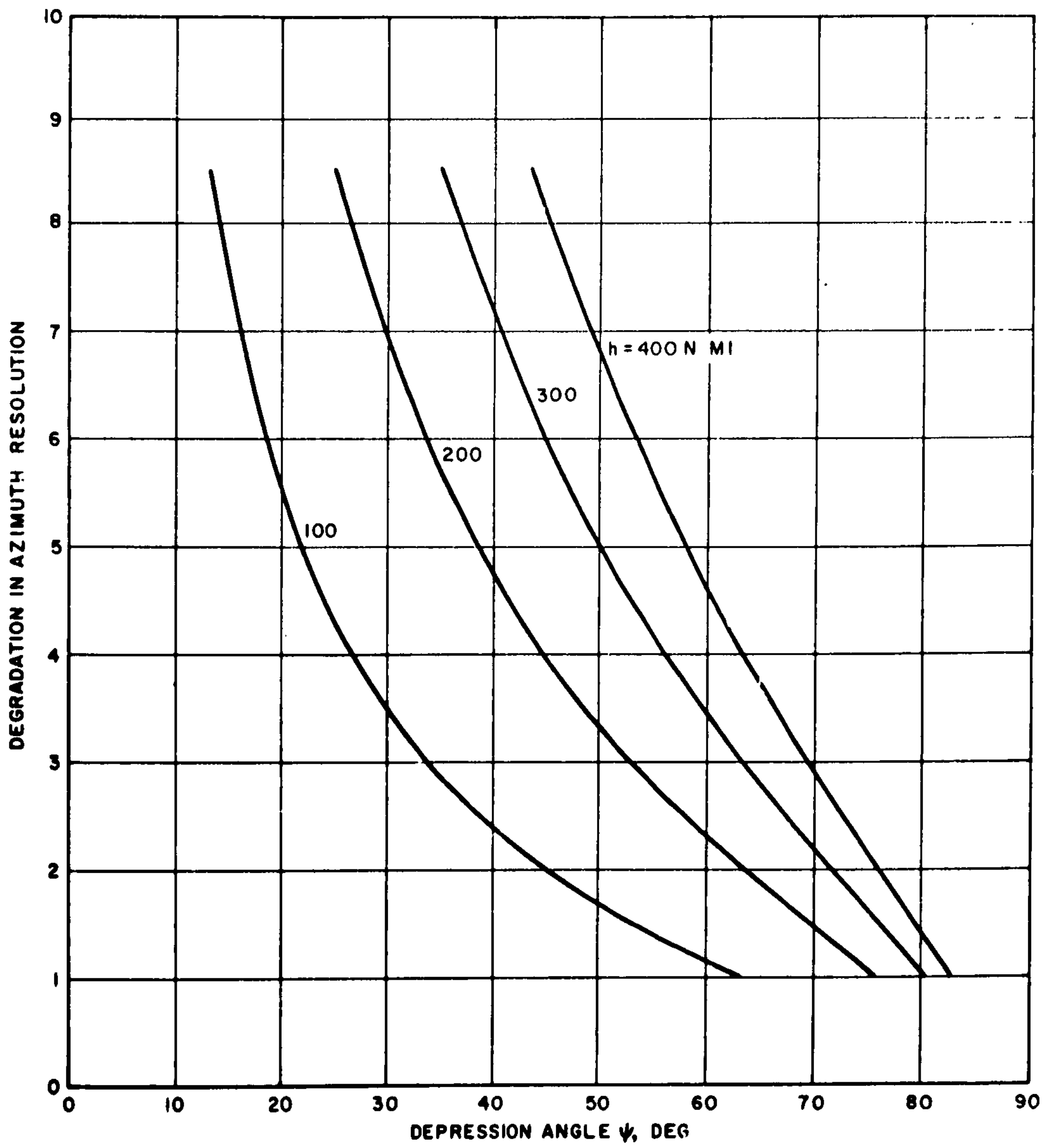

Figure 14. Degradation in Azimuth Resolution Due to Ground Range-Gating

Map-strip ΔR = 50 n.m.

$$\text{Degradation} = \frac{h}{\Delta R \tan \psi}$$

8.0 MULTIPLE CHANNEL RECORDING OF SYNTHETIC ARRAY DATA

Based on the concept that the final processing of a single azimuth element of synthetic array data requires only the radar returns from a fixed range, it would appear practical to segment the data in range and to record each range individually. The principal advantage of this type of recording would be a significant reduction in the base bandwidth required in the recording system.

Due to the fact that the range to a target over the build-up interval of a synthetic array is not actually fixed, the concept of multiple channel recording is only valid to the extent that the range variation is essentially constant within the resolutions of the system.

8.1 Commutation Recording

The concept of segmenting the range information in time and subsequent sampling is given in Figure 15.

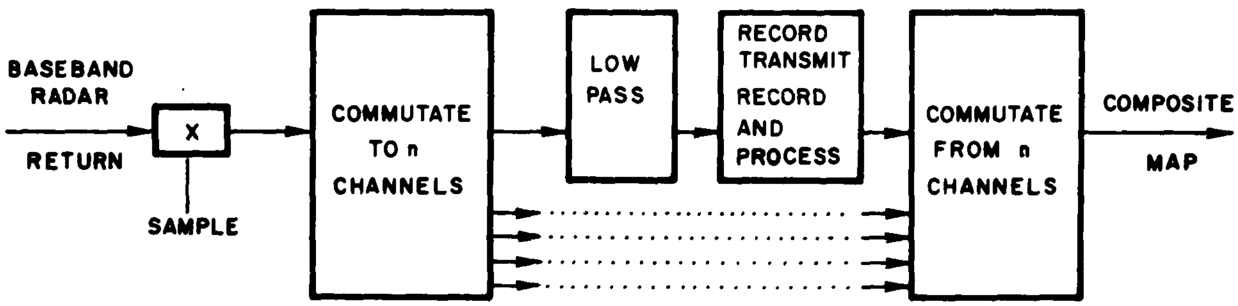

Figure 15. The Use of Sampling and Commutating to Produce Low Frequency Recording on Multiple Channels

The radar return is sampled at the range resolution rate $\frac{1}{\delta_T}$ where δ_T is the time associated with the range resolution δ_r. The base bandwidth W of the radar return is necessarily

$$W = \frac{1}{2\delta_T}$$

The samples are then commutated sequentially into n channels which effectively have a sampling rate of $\frac{1}{n\delta_\tau}$. In each channel the information is contained in a base bandwidth W_i where

$$W_i = \frac{1}{n} W \quad .$$

Subsequent processing of the channels individually requires that the initial phasing of the sampling and the commutation be directly related to the phase of the pulse repetition of the radar. Proper phasing would insure that each channel contains consistent range information. Since each channel is processed independently, interchannel stability is not necessary.

By commutation of the processed channels, a composite map can be constructed. Failure of any one of the channels would result in the composite map displaying regularly spaced streaks of width equal to the range resolution.

It would appear that any degree of resolution could be obtained with this method with a finite frequency recording capability. In the case where interchannel stability could be maintained, this would be possible by choosing a sufficiently large n. However, where the channels are processed individually, a lower bound to the resolution results from the necessity to process data from an interval of range data in order to produce a single resolution element.

8.2 Ranging on a Flat Earth

The geometry of Figure 16 indicates that the distance d from vehicle to target is

$$d = \sqrt{d_o^2 + x^2}$$

$$\approx d_o + \frac{1}{2}\frac{x^2}{d_o}$$

where the distance of closest approach d_o is

$$d_o^2 = h^2 + r^2$$

in relation to the altitude h and the range r.

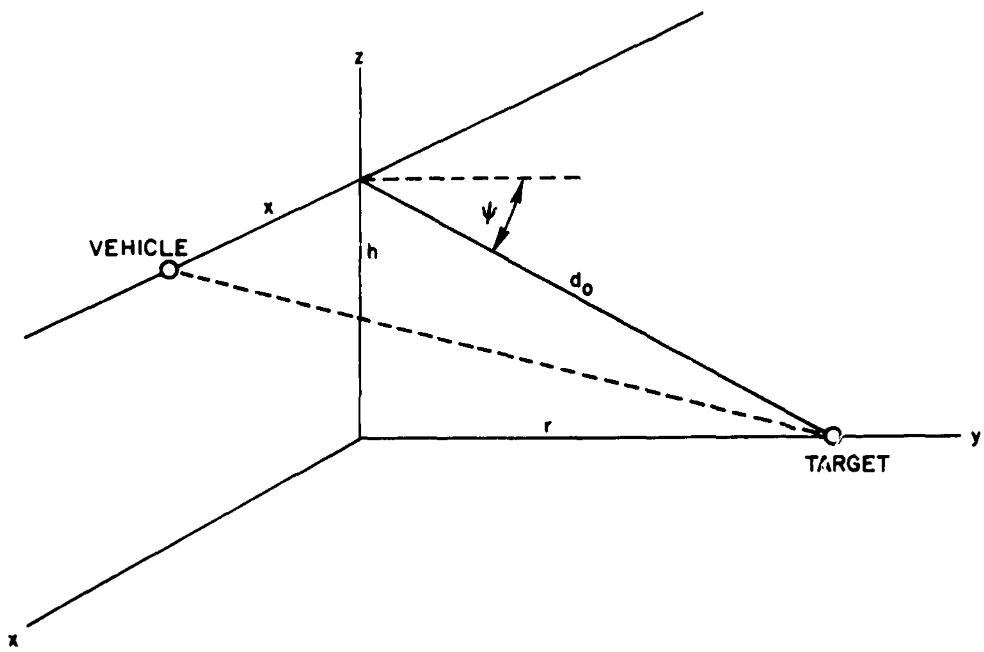

Figure 16. Geometry of Flat Earth Ranging

The range resolution δ_r is thus related to the variation in the distance of closest approach through the formula

$$\delta_{d_o} = \delta_r \frac{r}{d_o} = \delta_r \cos \psi$$

In order to keep the variation in range within one range resolution, it is necessary to restrict the variation $d-d_o$ to a value less than or equal to δ_{d_o}. Therefore,

$$d-d_o \leq \delta_{d_o}$$

or effectively

$$\frac{1}{2}\,\frac{x^2}{d_o} \leq \delta_r \cos\psi \quad .$$

The maximum value of x is bounded by the azimuth resolution

$$\frac{\lambda d_o}{4\delta_x} \leq |x| \quad .$$

The use of this bound and the relation

$$d_o = \frac{h}{\sin\psi}$$

produces the bound

$$\delta_r\,\delta_x{}^2 \geq \frac{\lambda^2 h}{16 \sin 2\psi}$$

where ψ is the depression angle.

A convenient expression is obtained by assuming equal azimuth and range resolution

$$\delta \geq \left(\frac{\lambda^2 h}{16}\right)^{1/3} .$$

A wavelength λ of 1/10 ft and an altitude of 16×10^5 ft (263 n mi) results in a 10 ft lower bound. Correspondingly higher bounds are obtained when a longer wavelength is used.

Since the bound is independent of velocity, a low flying aircraft has potentially a better capability than has a high altitude satellite. Three foot resolution is obtained at altitudes less than 43,200 ft with 1/10 ft wavelength.

8.3 Ranging on a Spherical Earth

The resolution bound of the preceding section is based on microscopic differences in ranging. For comparison purposes, it is instructive to obtain a corresponding bound to a circular orbit about a spherical earth. The geometry of Figure 17 indicates that the distance from the vehicle to the target is

$$\begin{aligned} d &= \sqrt{(h \cos \omega - R \cos \theta)^2 + h^2 \sin^2 \omega + R^2 \sin^2 \theta} \\ &= \sqrt{d_o^{\,2} + 2hR \cos \theta \, (1 - \cos \omega)} \\ &\approx d_o + \frac{1}{2} \, \frac{\dot{\omega}^2 hR \cos \theta}{d_o} \end{aligned}$$

where the altitude h is measured from the center of the earth and the distance of closest approach d_o is given by the relation

$$d_o^{\,2} = h^2 + R^2 - 2hR \cos \theta .$$

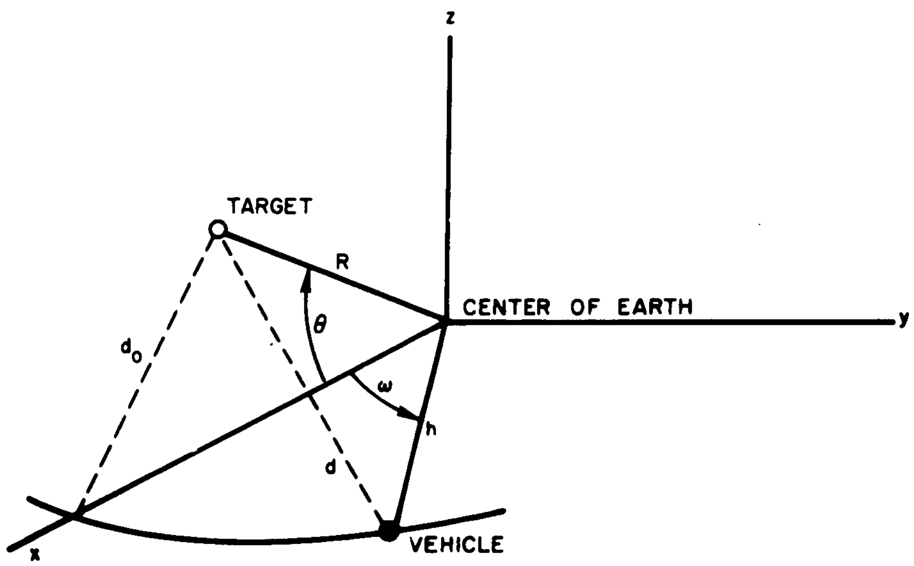

Figure 17. Geometry of Ranging from a Circular Orbit to Spherical Earth

The range resolution δ_r is then related to the variation in the distance of closest approach by the formula

$$\delta_{d_o} = \frac{hR \sin \theta}{d_o} \delta\theta$$

$$= \frac{h}{R} \delta_r \cos \psi$$

where the relations for range r,

$$r = R\theta \quad ,$$

and for the depression angle ψ

$$d_o \cos \psi = R \sin \theta \quad ,$$

are obtained from the geometry of the closest approach triangle of Figure 18.

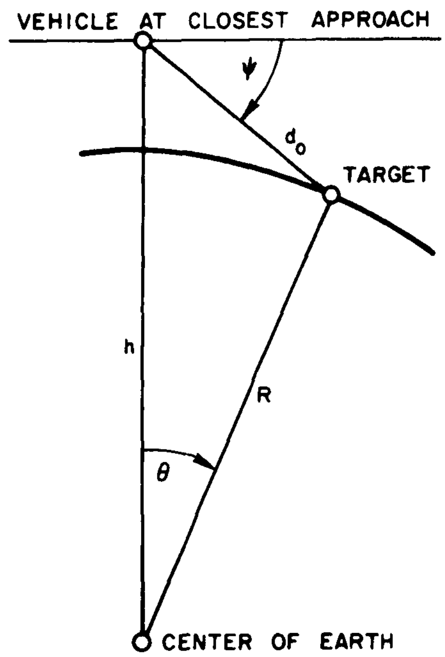

Figure 18. The Geometry of the Closest Approach

The bound on the range variation

$$d - d_o \leq \delta_{d_o}$$

produces the relation

$$\frac{\omega^2 hR \cos \theta}{2d_o} \leq \frac{h}{R} \delta_r \cos \psi \quad .$$

The value of angular travel ω is bounded by the azimuth resolution such that

$$\frac{\lambda d_o}{4h\delta_x} \leq |\omega| \quad .$$

The use of this bound with the preceding bound produces the bound

$$\delta_r \, \delta_x{}^2 \geq \frac{\lambda^2 \, (h - R\cos\theta)}{16 \sin 2\psi} \; \frac{R^2 \cos\theta}{h^2}$$

where the relation

$$d_o \sin\psi = h - R\cos\theta$$

is obtained from Figure 18. This is then the corresponding bound for a circular orbit about a spherical earth. Further simplification results from squaring the expression

$$R \sin\theta = d_o \cos\psi \quad ,$$

substituting it for $d_o{}^2$, and solving the resulting quadratic

$$\left[\frac{R}{h}\cos\theta\right]^2 - 2\cos^2\psi \left[\frac{R}{h}\cos\theta\right] + \cos^2\psi - \frac{R^2}{h^2}\sin^2\psi = 0$$

for $\frac{R}{h}\cos\theta$,

$$\frac{R}{h}\cos\theta = \cos^2\psi \pm \sqrt{\left(\frac{R^2}{h^2} - \cos^2\psi\right)\sin^2\psi}$$

where the + sign is of interest.

The use of this solution then produces the bound

$$\delta_r \delta_x^2 \geq \frac{\lambda^2 R}{16 \sin 2\psi} \left\{ \sin^2\psi - \sqrt{\left(\frac{R^2}{h^2} - \cos^2\psi\right) \sin^2\psi} \right\} \left\{ \cos^2\psi + \sqrt{\left(\frac{R^2}{h^2} - \cos^2\psi\right) \sin^2\psi} \right\}$$

This bound is plotted in Figure 19 for equal azimuth and range resolutions within the limits for which the depression angle ψ produces a ray intersecting the earth:

$$\frac{R}{h} \geq \cos\psi \quad .$$

The bound on allowable depression angles is plotted in Figure 20 as a function of altitude above the earth. At the terminator, the resolution bound is

$$\delta_x^2 \delta_r \geq \frac{\lambda^2}{32} \left(\frac{R}{h}\right)^2 \sqrt{h^2 - R^2} \quad .$$

In this case the use of a spherical earth produces essentially the same numerical results as a flat earth for low earth altitudes where small depression angles are excluded. As can be seen from the expressions, the use of a circular orbit and spherical earth introduces a decided degree of complexity over that involved with a flat earth model.

9.0 REDUNDANCY IN SYNTHETIC ARRAY DATA

A systematic consideration of a sequence of bounds indicates the constraints on a redundant operation of a synthetic array. Theoretically it is possible to operate a synthetic array in such a manner that the raw data bandwidth is identical to the processed data bandwidth. Under the assumption of equal input and output signal-to-noise ratios, the equal bandwidths indicate that there is no basic redundancy or excess noise in the raw data which can be removed by a processing technique. The degree to which the system is operated below the theoretical bounds represents the degree of redundancy in the raw data which can be suppressed by processing.

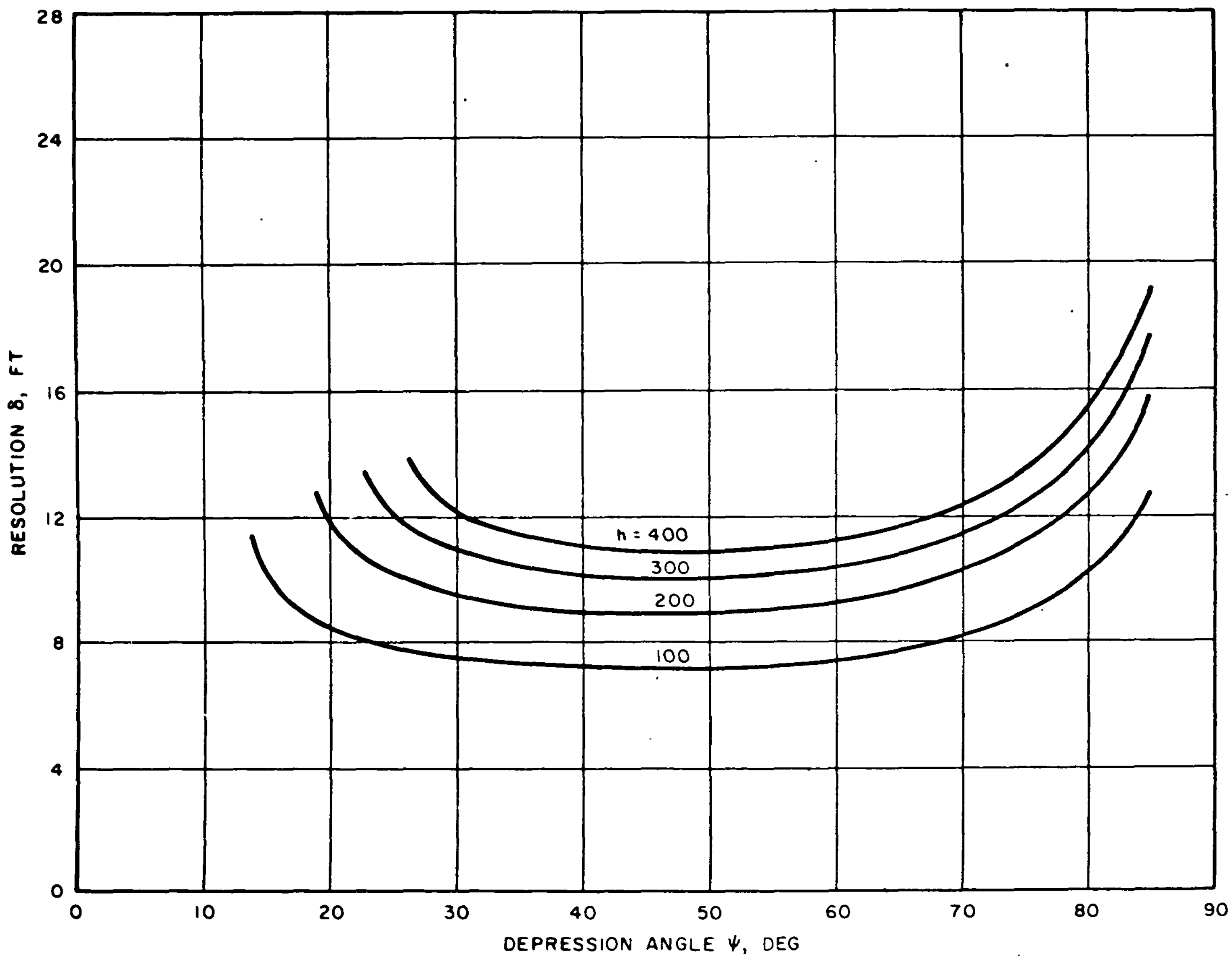

Figure 19. Independent Range Recording from a Spherical Earth

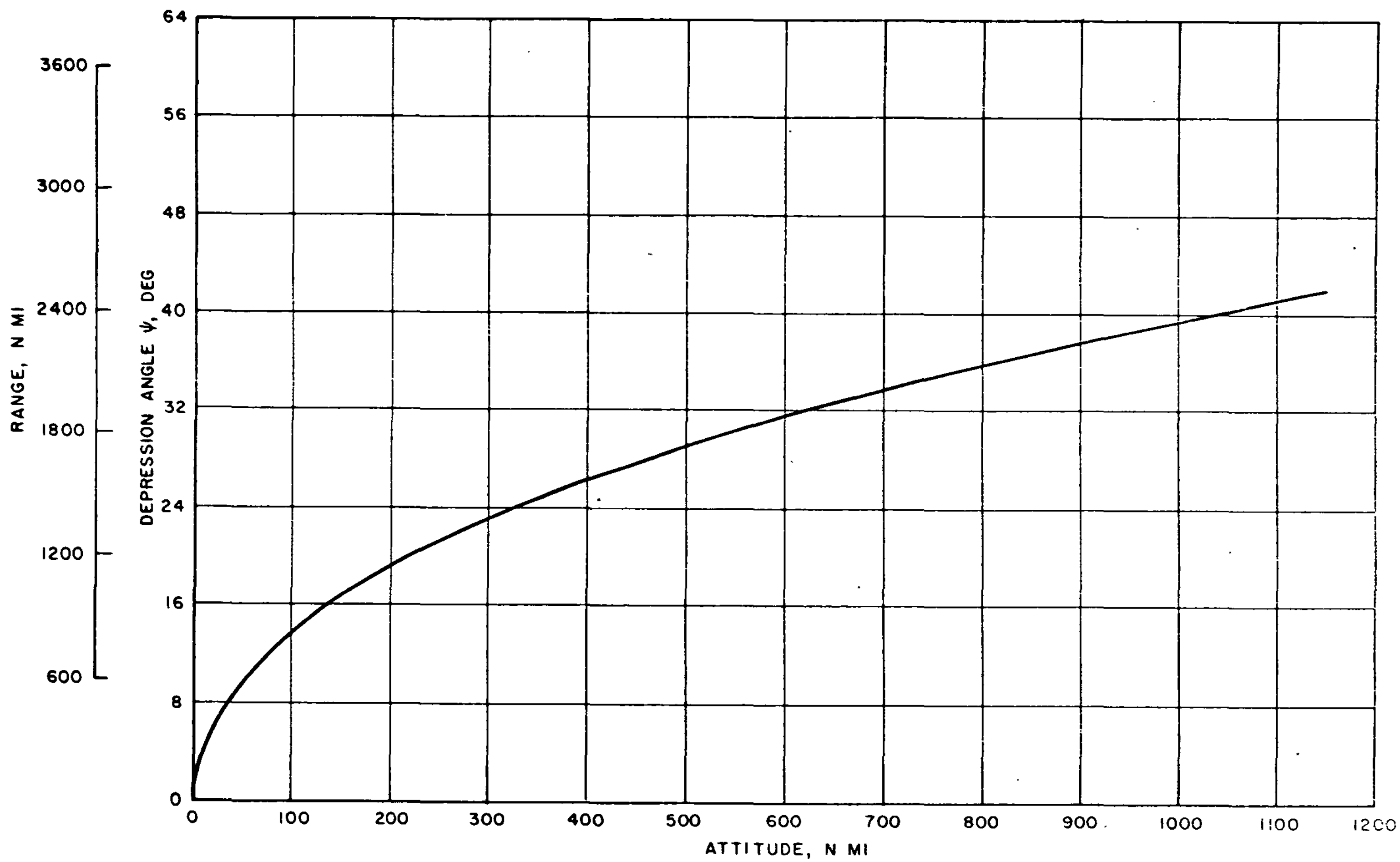

Figure 20. Depression Angle and Range of the Terminator

$$\cos \psi = \frac{R}{h} \qquad r = \psi R$$

9.1 Pulse Resolution Bandwidth

A linear system capable of passing a sequence of independent amplitudes spaced at interval δ_T apart is known to have a base bandwidth W_r equal to or greater than the reciprocal of twice the spacing δ_T:

$$W_r \geq \frac{1}{2\,\delta_T}$$

The bandwidth W_r is then the raw data bandwidth of a radar system using a pulse length δ_T to obtain a corresponding range resolution δ_r.

9.2 Ranging Resolution

The resolution δ_r in range is an upper bound on the pulse length δ_T. From the geometry of Figure 21 the bound is

$$\delta_T \leq \frac{2\,\delta_r \cos\psi}{c}$$

where c is the velocity of propagation and ψ is the depression angle.

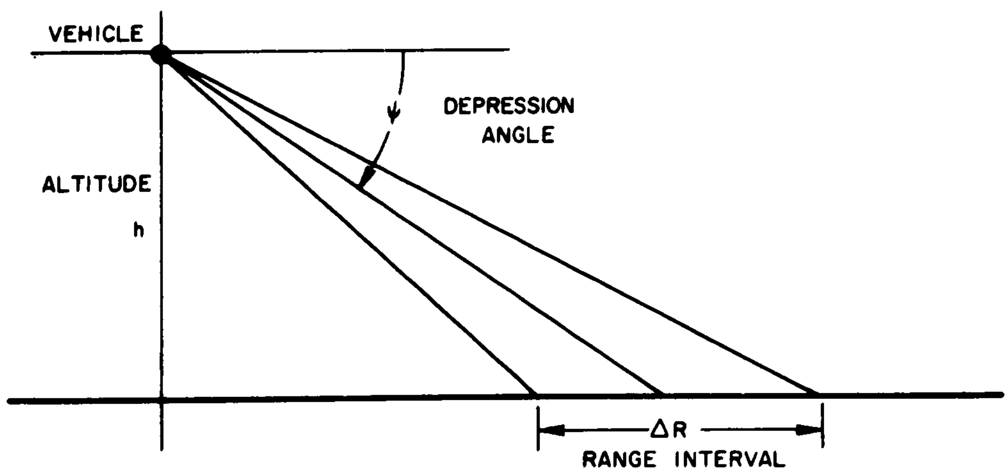

Figure 21. Range Geometry of a Synthetic Array

9.3 Interlacing Range Intervals

The time $\Delta\tau$ associated with the range interval ΔR of Figure 1 is

$$\Delta\tau = \frac{2\,\Delta R \cos\psi}{c} \quad .$$

By the appropriate interlace of the return echoes from the range interval ΔR, the pulse repetition frequency is bounded such that

$$\frac{1}{\Delta\tau} \geq \text{prf} \quad \text{or} \quad 1 \geq \Delta\tau\,(\text{prf}) = \frac{2\,\Delta R\,\text{prf}\cos\psi}{c}$$

9.4 Suppression of Synthetic Ambiguities

A bound on the azimuth beamwidth B_x of a synthetic array results from the necessity to suppress the first synthetic ambiguous beam occurring at an angle $\frac{\lambda}{2\Delta}$ from the center of the main synthetic beam:

$$\frac{\lambda}{2\Delta} \geq B_x$$

where Δ is the spacing of the elements of the array. In terms of the pulse repetition frequency (prf)

$$\text{prf} = \frac{v}{\Delta}$$

the ambiguity bound is

$$\text{prf} \geq \frac{2v}{\lambda}\,B_x \quad .$$

9.5 Doppler Spectrum

The doppler shift f_d of a target is approximately

$$f_d \approx \frac{2v}{\lambda} \theta$$

where θ is the angle of the target from the normal off the flight path. For a beamwidth B_x the high frequency doppler spectrum is then

$$\Delta f_d = \frac{2v}{\lambda} B_x \quad .$$

Sampling the inphase and quadrature components then requires by the sampling theorem that

$$\text{prf} \geq \Delta f_d = \frac{2v}{\lambda} B_x \quad .$$

This bound is identical to the ambiguity bound of the previous section.

9.6 Target Illumination

Over the build-up length L of the synthetic array data, it is necessary to illuminate a greater length X:

$$X \geq L \quad .$$

The illumination length X is given by the relation

$$X = RB_x$$

where R is the range. The build-up length L is related to the azimuth resolution δ_x through the synthetic beamwidth $B_s = \frac{\lambda}{2L}$ and the range R. It follows that

$$L = \frac{R\lambda}{2\delta_x} \quad .$$

The azimuth beamwidth B_x satisfies the illumination bound

$$B_x \geq \frac{\lambda}{2\delta_x}$$

9.7 Cascading Bounds

The base bandwidth W_r of the raw synthetic array data is bounded by the following sequence of bounds:

$W_r \geq \frac{1}{2\,\delta_\tau}$	Pulse resolution bound
$\frac{1}{\delta_\tau} \geq \frac{c}{2\,\delta_r \cos\psi}$	Range resolution bound
$1 \geq \frac{2\,\Delta R \text{ prf} \cos\psi}{c}$	Interlace bound
$\text{prf} \geq \frac{2v}{\lambda} B_x$	Ambiguity bound or Doppler bound
$B_x \geq \frac{\lambda}{2\,\delta_x}$	Illumination bound

implying

$$W_r \geq \frac{1}{2\,\delta_\tau} \geq \frac{c}{4\,\delta_r \cos\psi} \geq \frac{\Delta R \text{ prf}}{2\,\delta_r} \geq \frac{v\,\Delta R\,B_x}{\lambda\,\delta_r} \geq \frac{v\,\Delta R}{2\,\delta_x\,\delta_r}$$

9.8 Range Sampling

The number of samples in a range interval ΔR with a range resolution δ_r is $\Delta R/\delta_r$. This number is equal to the number of pulse lengths δ_τ in the interval of time $\Delta\tau$ allotted to the range interval ΔR:

$$\frac{\Delta\tau}{\delta_\tau} = \frac{\Delta R}{\delta_r} \quad .$$

Application of the interlace bound on the pulse repetition frequency prf

$$\frac{1}{\Delta\tau} \geq \text{prf}$$

implies the bound

$$\frac{1}{\delta_\tau} \geq \frac{\Delta R \text{ prf}}{\delta_r} \qquad \text{Range sampling bound}$$

This bound is identical to the cascade of the range resolution bound and the interlace bound.

9.9 Azimuth Sampling

The rate with which the azimuth resolution samples δ_x occur from a vehicle moving with velocity v is v/δ_x. It is then necessary that the sampling take place at a rate greater than the rate of occurrence

$$\text{prf} \geq \frac{v}{\delta_x} \qquad \text{Azimuth sampling bound}$$

This bound is identical to the cascade of the Ambiguity bound and the Illumination bound.

9.10 Processed Data

For an azimuth resolution δ_x and range resolution δ_r, the rate of data is

$$\text{Processed Data Rate} = \frac{v\ \Delta R}{\delta_x\ \delta_r}$$

for a velocity v and range interval ΔR. The base bandwidth W_p associated with this data is then

$$W_p = \frac{v\ \Delta R}{2\ \delta_x\ \delta_r}$$

The cascade of bounds indicates that the bandwidth W_r of the raw data is bounded by the bandwidth W_p of the processed data

$$W_r \geq W_p$$

These two bandwidths are equal when each of the bounds in the cascade is satisfied with an equality. In this condition it would appear that there are no fundamental redundancies in the system, implying that no processing of the raw data can reduce its quantity. On the other hand, where the bounds are not satisfied with an equality, the degree of redundancy in the raw data results from the degree of inequality in operating the system.

10.0 CONCLUSIONS

The operation, recording, and processing of synthetic array data presents a rather tightly constrained operation when its maximum capabilities are desired. The problem involved in recording is basically one of recording an amount of data equal to the data in the finished map. Where the data is segmented into multiple channel recording, limits arise on treating the channels independently. To go beyond these limits would require a limited degree of inter-channel stability. The concepts of pre-summing and range-gating appear to degrade the azimuth resolution. Since under maximum performance conditions the quantity of raw data is equal to the quantity of processed data, it would appear that a reduction in data by limiting the raw data would present similar degradation to the performance of the system.

The theoretical requirements on recording are thus fairly well set by the desired resolution and the width of the map strip.

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